BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 66 (70), Numărul 3, 2020
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

PHYSICAL PRINCIPLES IN REVEALING THE WORKING MECHANISMS OF BRAIN. PART III

BY

NICOLAE MAZILU[1,2,*]

[1]3707 Dauphin Dr, NE, Canton, OH 44721, USA
[2]Timotei Cipariu, #1, Focșani, Vrancea, 600004, Romania

Received: July 1, 2020
Accepted for publication: September 17, 2020

Abstract. The living brain is physically modeled as a universe, analgous to the existing physical model of the universe. While in the physical model the gravitation prevails, in the brain universe the electricity prevails, but the mathematical description we provide is essentially the same in both cases. What is imperiously necessary in this approach is, first, a metric description of matter, then, of course, the physical interpretation of this description. These issues were treated, in their essentials, in the previous two instalments of the work. The object of the present episode of the work is a classical space image of the matter, as described from a ‘central’ point of view. This image is connected with the concept of memory, for which we uphold the idea that in classical physics it has a well-known counterpart: the inertia. It is accomplished based on a general approach of the motion, suggested by generalizing Kepler’s classical model of motion.

Keywords: inertia; memory; matter density; Euclidean reference frame; torsion of space; Kepler’s laws; celestial matter; neurons.

[*]Corresponding author; *e-mail*: nicmazilu@gmail.com

1. By Way of Introduction: a Synopsis of Bygone Ideas

The series of works already publishedin this Bulletin [(Mazilu, 2019, 2020); these articles will be cited here as I and II respectively], or in the course of publication from now on, intends to fill an important gap in the physical science: a missing physical theory of brain. While any existing physical theory connected with this awe-inspiring modern subject of human knowledge is only incidental, so to speak, we start from an entirely different point of view, namely that the brain is a universe by itself. However, unlike any physical universe ever imagined by man, which, as a physical model, is exclusively mass-dominated, the universe connected with the description of brain is charge-dominated, at least as we can judge on account of the class of nondestructive experiments of medical interest. Consequently, this series of works has the task of discerning, first, the principles of physical description of a universe in general, and then of passing with a theory built on such principles, on to applications to a charge-dominated universe, particularly to the brain. The essential one among principles presented to our knowledge in this enterprise, seems to be that of *scale transition*, of which physics appears to take close notice in its applications lately, even to the point of fundamental applications (Nottale, 2011). From this point of view, the brain theory can be viewed as a theory of fundamental physical particles: both the universe of brain and that of physical particles are charge-dominated, even though at different space and time scales.

While, up to this point, the work has been dedicated to showing the technical needs of a cosmological theory in general, in order to serve in extracting the essentials in terms of the needs of modelling, starting with this episode and going onward, we concentrate on issues specifically connected with the brain. And it is even a matter of cursory observation that the basic issue of a universe that will explain the existence and overall function of the brain is connected with the *physical property of the memory*. While, due to our specific approach of the problem of brain, it is impossible to get over the necessity of revealing specific technicalities of a cosmology as usually acknowledged, from now on we shall have closer in sight only those mathematical needs that can be specifically connected to the physical description of the memory. Going a little ahead of us here, we can state, however, that the memory is the physical property of *any* material universe whatsoever. Only, historically speaking, it was disguised under different names. In fact, we do not know for sure but only about a single such name in physics: *the inertia*. From this point of view, the physics at large can borrow heavily itself, from the theory of brain, in order to add something positive,in general, to the existing natural philosophy.

The main physical point of the previous two contributions of this series was, we might say, the natural philosophical concordance of the structure of brain as a universe, with a structure of any universe imagined by man. We have insisted much on the two ideas which helped create the modern wave mechanics: that of Louis de Broglie and that of Erwin Schrödinger. This insistence is explainable by the specific features of our problem: if it is to have a 'mechanics' of the brain universe, this cannot be but a 'wave mechanics' at the most. However, inasmuch as the two fundamental ideas should be involved in the cosmology in general, we need to take notice that, in spite of their usual presentation, by and largeas antagonistic, there is a point of view from which they appear as identical. This is the point of view of scale transition, the one from which,contemplated, the wave mechanics itself appears as an essential science of the description of any universewhatsoever. This description takes the positive form of *interpretation*, which became the critical issue of the natural philosophy connected with the invention of the wave mechanics. And, as we see it, the wave mechanics is simply the necessary theory that completes the historical steps in the theory of light, culminating with the latter times'principle of asymptotic freedom in the theory of strong interactions. We believe that a short review of these historical steps will be beneficial fora proper understanding of the picture as a whole, still necessary in a suitable bridging of some theoretical gaps.

1.1. Brief History of Perception of the Light Phenomenon

The first physical image of light phenomenon was, we suppose, that of Thomas Hobbes: a global concept, as it were, involving the idea of "orb" (Hobbes, 1644). It was probably religiously inspired – as, in fact, any natural philosophical idea was – byan analogy with the heart. The analogy did not work quite properly: everyone whocan see the lightcan eventually get the idea that itis expanding only, never contracting, like the heart periodically does in fulfilling its job. However, the idea that light is *material* settled this issue in an unexpected way, insofar as, based on it, Robert Hooke placed the periodic motion where it should belong, just by a tight logic: it takes place within the 'orb' – read 'wave surface of light', in view of the later work of Augustin Fresnel – otherwise the light would be able destroy the transparent materials it penetrates, and such an event has never been noticed in human experience. The periodic motion is a 'pulse', more precisely an "orbicular pulse", if it is to use the Hooke's own words (Hooke, 1665). This last concept improves kinematically, even thoughit was actually taken mostly dynamically, upon Hobbes' purely geometrical "line of light", naturally filling in for the fact of expansion, by the idea of propagation of light along a direction. This was just about the first case in the Newtonian epoch, whereby the human spirit was starting filling the empty ideal world image provided by geometry, with properties of the world of human experience. The notorious – and scientifically typical, we should say – caseof this start of the classical natural philosophy in the epoch is, of course, the triad of Kepler laws, facilitating the invention of forces by Isaac Newton. Properly generalised, this case can still be taken as the epitome of such a procedure for any universe whatsoever, particularly for brain.

The phenomenology of light was limited, at the time of Newtonian inventions, to just two specific phenomena: *reflection* and *refraction* of light. Newton 'fortified', as it were, this phenomenology technologically (see his *Opticks*), in order to avoid the 'invention of hypotheses', by creating an experimental basis which, under different circumstances of course, works even today. However, based exclusively on that phenomenology, and therefore abundantly still 'inventing hypotheses', Hooke created a concept of *light ray* (*loc*. *cit*. *ante*, pp. 53 – 69), in which he incorporated what we think of as the first rational theory of colors. In this concept, the color is controlled by the angle between orbicular pulse and the mathematical rays delimiting a plane construction that can be rightfully called *physical ray*. It is on this concept, originally introduced by Thomas Hobbes, that the experimental basis created by Newton helped improve, by adding one important differentia to it, on which we have to abide for a while, for it is of essence in what we have to say here. Thomas Hobbes insists upon the fact that *the physical ray is not a plane figure*, but *a solid one*, a cone or a cylinder, or even a more general *tube*. Quoting:

PROPOSITION IV

The ray is a solid space

> Since a ray is the path through which a motion is projected from a luminous object and there can be no motion except of a body, it follows that a ray is the place of a body and therefore has three dimensions. Therefore, a ray is a solid space.
>
> *Definition of direct and refracted rays*
>
> A *direct* ray is the one whose section by a plane passing through its axis, is a parallelogram…
>
> A *refracted* ray is the one composed of two direct rays making an angle along an intermediate part…
>
> *Definition of the line of light:*
>
> The line where the sides of a ray begin… [(Hobbes, 1644), *our translation; see also* (Shapiro, 1973), *and the Portuguese translation of* Tractatus Opticus *in* Scientiae Studia, Sao Paulo, Vol. **14**(2), pp. 483 – 526 (2016)].

The whole classical discussion of optics before Newton is always done on that 'section by the plane through axis', geometrically defining the physical ray to Hobbes, where the implicit assumption – no doubtin our opinion – is that the 'section' thus defined is 'generic': there is no distinguished axial section of the ray. Hooke's definition of the colors does not satisfy this requirement. This was the conclusion of the Newton's celebrated, detailed and careful experiments with a prism, or a set of prisms, which proved that the color is a property of light, varying indeed directionally, but as a feature of *homogeneity* of the ray [*see his* Opticks; *see also* (Shapiro, 1973)]: different homogeneous rays are distributed in a certain direction across the spectrum. The theory created by Newton extends the experimental observation that different homogeneous rays have different refrangibilities, corresponding to different colors, so that the homogeneity needs to be further characterized by a specific differentia: the color. In other words, from the point of view of the color, the ray geometry cannot be plane, as the Hooke's theory would imply. In Newton's vision, a homogeneous ray still satisfies the Hobbes' definition, and has to be described by a solid geometry: it is a collection of homogeneous rays – the spectrum – that only incidentally, *i.e*. depending on the presence of the prisms, extends directionally in the cross-sectional plane.

It should be illuminating, we believe, especially in understanding the concept of physical ray in general, to notice that it was eventually discovered that the color must be described by a gauge group acting in the cross-section of the ray (Resnikoff, 1974), reproducing a group action of SL(2,R) type, and that Erwin Schrödinger pioneered this very discovery (Schrödinger, 1920). One can rightfully say that, with the theory of colors he created in 1920, Schrödinger was in fact completing an 'apprenticeship' for the physics which he started building six years later (Mazilu *et al*., 2019). Be it as it may, what we think is worth retaining –in the spirit of today's physics, of course – from the brief history we just presented, is the fact that the epoch of reflection and refraction phenomenology produced the concept of a *light ray as a tube*, having the color as *a transversal property of homogeneity, described at present by a gauge group*.

Along this historical path, Augustin Fresnel started a new epoch in phenomenology, marked by the introduction of *diffraction* of light as a new phenomenon, in the description of which the periodic properties of light were the usual observables, and thereby the wave nature of light came closer to our rational understanding. The obvious spatial periodical pattern in the *recordings* of diffraction phenomena could thus be explained physically, as a mechanical interference phenomenon. In so doing, the optics made reference to the harmonic oscillator, in order to understand the intensity of light for instance, to say nothing of some other physically fundamental necessities, like the very definition of the *intensity of light*. However, this reference is, by stretching a little the meaning of words, 'illegal', to say the least, in the case of light, inasmuch as the light phenomenon is a far cry from exhibiting the *inertial properties* required by a proper dynamics of the harmonic oscillator. One can even say that this is the deep reason the ideas of Fresnel encountered a firm opposition from Laplace and Poisson, who were abiding by the mathematical rules of classical dynamics [(Fresnel, 1821, 1826) and the work cited therein]. Indeed, as a purely dynamical system, the harmonic oscillator is a system described by forces proportional to displacements (those type of elastic forces, used initially by Hooke to explain the behavior of light), and in the case of physical optics the second principle of dynamics is quite incidental, as it were. It was introduced only by a natural mathematical property of transcendence of the second order ordinary differential equation: it describes *any* type of periodic processes. However, the fact is that in the foundations of modern physical optics, the periodic processes of diffraction have more to do with the theory of statistics than with the classical dynamics (Fresnel, 1827). That much was obvious from the very beginnings of the modern optics, and we shall come back to these issues here, for the specific case of electric charge.

This is, however, not to say that the harmonic oscillator is to be abandoned altogether, as a model, because this is not the case, either from experimental point of view, or even theoretically, as we also will show here in due time. All we want to say is that we need to find its right place and form of expression in the theory, and those are indicated, again, through the order imposed by the measure of things, this time as their mass. Indeed, dynamically considered, the second order differential equation,is an expression of the principle of inertia, and involves a *finite* mass. On the other hand, for light the mass is practically nonexistent. However, if the second order differential equation is imposed by adding the diffraction to the phenomenology of light, this means that such an equation actually describes *a transcendence between finite and infrafinite scales of mass* [(Georgescu-Roegen, 1971); for a closer description of the concept, one can also consult (Mazilu *et al*., 2019)]. As, again, we shall see later here, the mathematics of scale transitions between finite and infrafinite – in our case here, infinitesimal – gauges in a scale relativity, fully respects the rules related to the harmonic oscillator model. In fact, the whole wave mechanics, as a science, can be constructed based on such rules, which appear to be universal.

Now, along with the settling of Fresnel’s theory in physical optics, a few changes in the natural philosophy have been taking place. First in the order of things changed, was making the dynamics ‘lawful’, as it were, in the case of light. The first step was *to identify the phase*, mathematically involved as an independent variable in the trigonometric functions describing the diffraction in the light phenomenon, *with the time of an evolution*: according to Fresnel’s theory of diffraction, the phase had to be linear in time. A condition which brought *the frequency* front and center, and with it the concept of *wavelength*, thus generating right away a whole new experimental technology of the Newtonian kind, leaving nevertheless behind what the dynamical principle really needed for a sound physical theory: first, the *elastic properties* of the medium supporting the light and, secondly, the *interpretation of light*, which obviously required the old idea of particle, and therefore the inescapable inertial properties. The Fresnel’s ellipsoid of elasticities pretty much fills in for the first aspect of this issue, while the second one was delayed, if not flatly left in suspension ever since, being occasionally replaced with *ad hoc* creations of mind, and so is it, actually, even today to a large extent. A proper dynamical use of the second principle of dynamics in the matters of light came in handy only later on, with the advent of the electromagnetic theory of light. This theory of light has in common with the old Fresnel optics the one equation that models the space and time periodical properties no matter of their physical approach, as long as these properties are described by a frequency: *the D'Alembert equation*.

This was the point where Louis de Broglie entered the stage of light physics [(de Broglie, 1926); see I, §2.2]. Based on his association of a wave with a material point, which thus became a 'wave phenomenon', de Broglie used the classical concept of light ray, as this was left by Newton and Hooke, for an interpretation of the light, according to the precepts of the wave mechanics. As we have duly noticed (*loc*. *cit*. *ante*), in the process of interpretation de Broglie was obligated to complete the very concept of light ray with differentiae above and beyond those introduced by Fresnel, in order to prove that the interpretation does not contradict the phenomenon of diffraction as perceived by us: the diffraction is a phenomenon that can be described wave-mechanically, when proper physical interpretation is adopted, or simply mechanically, when a proper physics of waves is adopted. In hindsight, one must notice that this was, in fact, the whole point of the Fresnel's theory, in order to be able to rightfully conclude the Huygens' global image of light, and to overcome the reproaches of Laplace and Poisson. This also bestows a right physical character to the optical theory of light, along with the wave-mechanical theory of particles. However, the main offspring of this way of physics still awaits to be clearly recognized, for, in fact, the de Broglie's theory of the light ray meant an addition to the very phenomenology of light: there is a *fourth* phenomenon to be added to this phenomenology, and this is the *holography* (see I, §2.3 and §3.4). It came to be recognized as such only much later than the time of establishing of the wave mechanics, but only due to a particular manner of introduction of the wave function in the process of interpretation, which was duly respected by de Broglie on the occasion of his interpretation.

1.2. Specific Task: Addition to the Phenomenology

If one wonders why do we insist so much upon a methodical concept of light ray, the answer should not be quite out of hand: as we have shown previously (see I, §1.2 and II §§9 & 10), there should be an equivalence between a neuron and a light ray. The analogy can be taken even to details: the transport of light along a light ray is like the transport of electric charge along an axon, or vice versa. From this point of view, *viz*. of analogy, we can even say that the creation of electricity – apparently essential in the case of neuron – should be equivalent to the creation of light, an idea that can be turned into a basis of physical modeling of the synaptic connection. However, as, again, we have shown in the previous instalments of this work, in our opinion, the main point of the physics of brain should be in building a physics based on an idea by Karl Pribram, whereby the brain should be holographically modeled (Pribram, 2007). Only, we have to add that the holography takes here a special concrete shape that asks for a wave-mechanical approach of the physics of brain: the potential describing a physical structure is everywhere determined by the amplitude of the signal propagated along the ray [see I, §2.3, equation (2.19)]. And when it comes to the propagation of the charge, this amplitude can only described by nonlinear equations of solitonic type (see II, §5)

The key of this model of neurons as physical rays, would be, therefore, replacing the motion with a solitonic propagation. Indeed, one can hardly assume that the transmission of electricity inside the brain can be labeled as a motion in the dynamical or kinematical sense of the word. In fact, the economy of electricity has to have here a fundamental management existing, apparently, in no other universe, at least at the first sight. In other words, the phenomenology has to account for a strange phenomenon: *the interaction at distance between the rays*. In optics this property appears as the coherence of the rays. This can be best understood from the following words of renowned neurologist Karl Lashley, as quoted in the 1998 work of Karl Pribram:

> Here is the dilemma. *Nerve impulses are transmitted over definite, restricted paths* in the sensory and motor nerves and in the central nervous system from cell to cell through definite intercellular connections. Yet *all behavior seems to be determined by masses of excitations*, by the form or relations or proportions of excitation *within general fields of activity*, *without regard to particular nerve cells*. It is *the pattern, and not the element that counts*. *What sort of nervous organization might be capable of responding to a pattern of excitation without limited specialized paths of conduction*? The problem is almost universal in the activities of the nervous system and some *hypothesis* is needed to direct further research [(Pribram, 1998); *our emphasis*].

We translate this problem as it was suggested in Pribram's own work, with the benefit of our detailed de Broglie model of ray, turned into a universal model: through the brain *the impulses are transmitted not only along rays, but also from ray to ray when necessary*. This aspect of propagation is virtually missing in any theory of rays – be it classical or quantal – but is surely felt as imperiously necessary in any such theory: it should be, as in the case of light itself, the basis of a firm definition of *coherence*. The connection between the brain locations of specific memories seems to depend on such a direct connection, independent of the paths of transmission inside the brain. Then, what is the relationship between the propagation and coherence? We present here a natural mathematical hypothesis, in the spirit of Karl Lashley, 'to direct further research', and develop, up to a point, its further consequences. Finally, among the things necessary to a theory, appears to be the *general meaning of electromagnetics* and its delimitation for the case of brain. It will be shown right away that a Yang-Mills generalization of electromagnetism will do, as in the case of static fields (see II, §10). This line shall be pursued here, as a gauging procedure, thus encompassing the idea that the Yang-Mills fields are able to include any properties of the nerve impulses, not only those centered around the electrical properties of these impulses (Drukarch *et al*., 2018).

The manner Schrödinger introduced his wave function uses, from a certain point onward, a variational principle formally identical to that used in theoretical physics for introduction of the complex potential in the theory of general relativity (Ernst, 1968, 1971). The most general condition defining the *Ernst potential* is that of stationarity of the spacetime metric, which means independence of the entries of the metric tensor of spacetime of the time coordinate. One can say that this fact is to be taken as the true original contribution of the general relativity in the physical explanation of the gravitation. Indeed, if the gravitation in general relativity is representable by the metric tensor, the property of stationarity means that it is independent of the order of events within the space described by this tensor. In this context the Carlton Frederick's idea that the wave function would have to be found among the components of the metric tensor (Frederick, 1976), would mean that the wave function, just like the gravitation itself, is independent of any history of the events in a given space.

This last idea is, nevertheless, very old: in fact it has been started by Newton himself when he invented his forces. These are the contemporaneous expression of the history embodied into a permanent motion, to wit, the Kepler motion, which thus can be viewed as an *expression of memory*, and this is what we shall do here. Now, to the extent where we succeeded in documenting this conclusion, it was started, from the side of the wave mechanics involved here, with the idea of memory,reached, however, 'negatively' as it were,by Edwin Crawford Kemble. Specifically, Kemble has an interpretation of the wave function – as this is understood in the wave mechanics, *i.e*. according to the definition of Charles Galton Darwin – for which, however, he was compelled to bring on stage the essential characteristic of an ensemble, *viz*. that of randomness (Kemble, 1937): *it acts as a history-destroying device*, a device that classical dynamics missed in its constructions, from obvious reasons, of course. Actually, it is this fact that justifies the Schrödinger's approach to wave mechanics, suggesting it as a natural and universal variant to Newton's dynamics. And this is, actually,one of the reasons we chose the wave mechanics as a universal description, independent of dynamical precepts, for which the classical dynamics appears as just a particular case. Quoting from Kemble:

> Experiment shows that the statistical properties of a large assemblage of independent identical microscopic, or macroscopie, systems (*i.e.*, a Gibbsian assemblage) which has been “aged” in a thermostat *at a definite temperature* T *for a sufficient length of time* usually become constant and independent of the initial state of the assemblage. The ultimate state is then defined to be *one of thermodynamic equilibrium at the temperature* T. By erasing all vestiges of the initial state *the thermostat acts as a history-destroying device*. To be sure there are numerous cases in which this function is imperfectly performed. In such cases the state of true thermodynamic equilibrium, or maximum entropy, is *not reached in any measurable time at moderate temperatures*. We may restrict the discussion for the present, however, to systems for which thermodynamic equilibrium is actually attainable [(Kemble, 1937), p. 433; *our Italics*].

These observations of Edwin C. Kemble regarding the definition of an ensemble serving to interpretation, not only accredit the idea that the wave function would have to be connected with the metric tensor of the stationary gravitation – that, we may add, gets a modern contour by the concept of *protective measurements* in the quantum mechanics (Aharonov *et al*., 1993) – but even the idea that,in the absence of matter, the spacetime itself would have to be stochastic.

These conclusions will be, by and large, our program of construction of the theory of brain. And the obvious first step seems for us an understanding of the cosmological perception of the matter. This has two fundamental aspects, from a mathematical point of view: the *image of space* from the perspective of a central observer – which is the only perspective the mankind can rightfully assume, the rest being just hypothesis – and the *manifestation of matter* from this perspective. These two topics are the object of what follows from the present instalment of our work.

2. The Classical Celestial Vault

It is probably worth consideringa little closer a simple observation on the scientific findings of the day: the different manners of human perception, be it direct or technologicallyassisted, do not indicate, when the concept of matter happens to be considered, the same physical propertiesfor the same region of space, if perceived from different physical perspectives. To wit, there are, for instance, discrepancies between the images of a region of the sky as seen in infrared say, and by regular, radio or X-ray telescopes. As these discrepancies turn always into paradoxes of our thinking – the missing matter, the dark matter, etc – a mathematical expression of the observations seems necessary, because the difference in perceptions compels us to a preliminary sound view in order to bring up-to-date the identity of the matter in a region of space.

The problem is that we hardly can define what 'sound view' may mean in general. However, we always can turn to the usual idea that different possibilities of observation are connected to different physical magnitudes that characterize the matter: the motion is connected to gravitational mass and charges, the light is connected to charges and motion, etc. A 'sound view', therefore, would be a fair mathematical description of the observations from different points of view, followed then by a connection of these mathematical descriptions, that can explain facts related to different universes at different space scales. For the problem in hand – *viz*. the physics of brain – we have facts related to brain, that are gathered by means entirely analogous with the facts related to the universe at large: *nondestructively*. We cannot touch the brain, and we cannot touch the universe. From different reasons, is true – touching brain would mean destroying it, while touching the universe seems out of question – but the fact remains fact. However, we can observe both of them by their functionality, inasmuch as they produce something accessible to our observation. And, insofaras in the case of the universe at large we have a pretty good image of these productions, let us describe them in such a way that the space scale may be obvious, in order to see if they do not have an expression of existence at different space scales.

2.1. The Geometry from a Point of View Located under the Canopy

The most realistic reference frame one can claim for any kind of observation cannot be but a limited portion of the Earth surface. Whether or not formally recognized, our experience and,implicitly, its physical explanation, of course,was inherently constructed under this universal circumstance. In our opinion, this circumstance needs to be taken in consideration in any realistic construction of a cosmology, for it influences any image we make of the universe. The science of geophysics, for instance, can teach us that the position of the reference frame depends on the time of tectonics (see II, §9). This is, however, a conclusion that depends heavily on a well-groomed, if we may say so, mathematical theory, as the example just cited plainly shows. Historically, though, the things went apparently in a simpler way: the realization of vastness and eternity of the universe forced us to the direct conclusion that *we are located in a point in space*. It is starting from this idea – heavilycorrupted by thinking,of course, for it copiously involves the imagination – that the whole science was built. One product of this imagination is the geometry of space, a form of which we shall present here, but with the obligation in mind, to fill in later for the reality from which it started, which thereby takes a concrete form: the idea of *finiteness*.

The Euclidean geometry of a three-dimensional space referred to a certain point, can be written in the language of position vectors, which, in a reference frame with its origin in the point, can be written in the form

$$\boldsymbol{r} = r \cdot \hat{\boldsymbol{e}}_r \tag{2.1.1}$$

Here r is a length assigned to the position vector of a point in such a reference frame, and $\hat{\boldsymbol{e}}_r$ is the unit vector orienting this length, in order to make a vector out of it. Now, in equation (2.1.1) we adopted, in fact, a spherical polar coordinate system adapted to the reference frame in which this very equation is written. Therefore, in a matrix form, using implicitly a universal Cartesian referrence frame, the equation (2.1.1) should be written in the form

$$\boldsymbol{r} = |x\rangle \equiv \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \quad \hat{\boldsymbol{e}}_r \equiv \begin{pmatrix} sin\theta cos\varphi \\ sin\theta sin\varphi \\ cos\theta \end{pmatrix} \tag{2.1.2}$$

In most of the applications connected to the physics of matter in space, like, for instance, in the kinematicalproblems, only the first- and second-order *symmetric* differentials are of importance, ever since the Newtonian natural philosophy was instated in science. The first order differential of the position vector, which is always ‘symmetric’ by its very nature, can be calculated directly from (2.1.1), using the usual differentiation rules:

$$d\boldsymbol{r} = (dr) \cdot \hat{\boldsymbol{e}}_r + r \cdot (d\hat{\boldsymbol{e}}_r) \tag{2.1.3}$$

In order to calculate the differential of the unit vector orienting the position vector, we use the second of the definitions in (2.1.2), with the result

$$d\hat{\boldsymbol{e}}_r = (d\theta) \cdot \hat{\boldsymbol{e}}_\theta + (sin\theta d\varphi) \cdot \hat{\boldsymbol{e}}_\varphi \tag{2.1.4}$$

where the two new unit vectors from the right hand side here are given by the matrices

$$\hat{\boldsymbol{e}}_\theta \equiv \begin{pmatrix} cos\theta cos\varphi \\ cos\theta sin\varphi \\ -sin\theta \end{pmatrix}; \quad \hat{\boldsymbol{e}}_\varphi \equiv \begin{pmatrix} -sin\varphi \\ cos\varphi \\ 0 \end{pmatrix} \tag{2.1.5}$$

These two unit vectors, together with the one defined in (2.1.2) form an Euclidean reference frame, referred to spherical polar coordinates, the one in which the vector $\boldsymbol{r}$ is defined as the matrix $|x\rangle$ from equation (2.1.2). Using (2.1.4) in (2.1.3), the differential of the position vector becomes:

$$d\boldsymbol{r} = (dr) \cdot \hat{\boldsymbol{e}}_r + (rd\theta) \cdot \hat{\boldsymbol{e}}_\theta + (rsin\theta d\varphi) \cdot \hat{\boldsymbol{e}}_\varphi \tag{2.1.6}$$

The square of this differential of the position vector is given by the inner product:

$$d\boldsymbol{r} \cdot d\boldsymbol{r} = (dr)^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2) \equiv (dr)^2 + r^2(d\Omega)^2 \tag{2.1.7}$$

with an obvious definition for $(d\Omega)^2$. We recognize here the regular Euclidean metric, written in polar spherical coordinates. One can verify the Frenet-Serret equations, describing the variation of Euclidean frame of the above unit vectors associated with the spherical coordinate system:

$$\begin{pmatrix} d\hat{\boldsymbol{e}}_r \\ d\hat{\boldsymbol{e}}_\theta \\ d\hat{\boldsymbol{e}}_\varphi \end{pmatrix} = \begin{pmatrix} 0 & d\theta & sin\theta d\varphi \\ -d\theta & 0 & cos\theta d\varphi \\ -sin\theta d\varphi & -cos\theta d\varphi & 0 \end{pmatrix} \cdot \begin{pmatrix} \hat{\boldsymbol{e}}_r \\ \hat{\boldsymbol{e}}_\theta \\ \hat{\boldsymbol{e}}_\varphi \end{pmatrix} \qquad (2.1.8)$$

This equation helps in establishing the *second symmetric differential* of the position vector used in calculating the accelerations. Indeed, from equations (2.1.6) and (2.1.8) we have, by the rules of differentiation:

$$\begin{aligned} d^2\boldsymbol{r} = \left(d^2r - rd\Omega^2\right) \cdot \hat{\boldsymbol{e}}_r \\ + (rd^2\theta + 2drd\theta - rsin\theta cos\theta d\varphi^2) \cdot \hat{\boldsymbol{e}}_\theta \\ +(rsin\theta d^2\varphi + 2sin\theta drd\varphi + 2rcos\theta d\theta d\varphi) \cdot \hat{\boldsymbol{e}}_\varphi \end{aligned} \qquad (2.1.9)$$

In the system of Newtonian dynamics, this vector represents the acceleration if, of course, it is referred to an adequate continuity parameter, playing the part of the time of problem. Right now, we do not proceed like that, but will discuss the general case of the differentials. However, by abusing a little of the classical terminology, the components of the vector (2.1.9) will still be designated as ‘accelerations’, just as the components of vector (2.1.6) will be called ‘velocities’.

In the case of classical free particle, the components of acceleration vanish, a condition that comes down to a system of three differential equations:

$$\begin{gathered} d^2r - rd\Omega^2 = 0 \\ rd^2\theta + 2drd\theta - rsin\theta cos\theta d\varphi^2 = 0 \\ rsin\theta d^2\varphi + 2sin\theta drd\varphi + 2rcos\theta d\theta d\varphi = 0 \end{gathered} \qquad (2.1.10)$$

This system can be solved by starting with the quadratic form $d\Omega^2$, which depends only on angles, and is needed in the first of the equations for the description of the second differential of the radial coordinate. It satisfies a simple differential equation; first, we have by direct differentiation:

$$d\left(d\Omega^2\right) = 2(d\theta d^2\theta + sin\theta cos\theta d\theta d\varphi^2 + sin^2\theta d\varphi d^2\varphi) \qquad (2.1.11)$$

Now, using here the last two equations from (2.1.10) we can produce the result

$$rd\left(d\Omega^2\right) + 4dr\left(d\Omega^2\right) = 0 \qquad (2.1.12)$$

and thus we can get a solution for the differential quadratic form $d\Omega^2$:

$$d\Omega^2 = \frac{c^4}{r^4} dt^2 \qquad (2.1.13)$$

Here c^2 is a constant having the dimensions of anarea rate, so that, if the continuity parameter t is time, the whole differential expression from the right hand side of this equation should be dimensionless, as the arclength of the unit sphere always is. In other words, we just have introduced, apparently only for convenience, the ‘time parameter’ t, to be measured by the metric of the unit sphere, in common use for expressing the celestial observations. Digressing a little: the equation (2.1.13) can be taken in the reverse, in the sense that it defines the spherical angle Ω itself – the continuity parameter on the unit sphere – *as a time parameter*. This is a fact known for ages in human history: the social time itself is measured by the positions of stars and orbs on the celestial canopy. It is the revision of this time that once triggered what is known as the Copernican revolution!

Now, coming back to geometry, inserting the result (2.1.13) into the first equation (2.1.10) we get

$$d^2r = \frac{c^4}{r^3}dt^2 \tag{2.1.14}$$

This equation, most commonly called the *Ermakov-Pinney equation* in physics, can be solved to give the known solution:

$$r^2 = At^2 + 2Bt + C \tag{2.1.15}$$

with A, B and C constants satisfying to the constraint:

$$c^4 \equiv AC - B^2 \tag{2.1.16}$$

In the words of a classical view: the radial coordinate is the coordinate of a *classical free particle*, provided the Kepler’s area law is verified. That law is crucial: inasmuch as it is verified for the Kepler motion, just the way it is verified for a free particle, that would mean that the two are somehow equivalent. The concept of interpretation by ensembles of free particles (see I, §2.3) indicates thatsuch equivalence should be approached from a wave-mechanical point of view. However, the concept of ‘freedom’ is a lot deeper than this first mark of interpretation, and this depth will be obvious as we go along with the work.

Now, again, a little digression on the previous results seems necessary, in order to justify our mention that we have to do here with the area law, and also some incidental further proceedings, as far as they may involve stochastic or fractal applications. Notice that the equation (2.1.13) is a direct consequence of the last two equations (2.1.10). They can be formally integrated as follows: first we have them properly arranged in the form of a homogeneous differential system having a skew-symmetric matrix

$$d\begin{pmatrix} r^2 d\theta \\ r^2 sin\theta d\varphi \end{pmatrix} = \begin{pmatrix} 0 & cos\theta d\varphi \\ -cos\theta d\varphi & 0 \end{pmatrix}\begin{pmatrix} r^2 d\theta \\ r^2 sin\theta d\varphi \end{pmatrix} \tag{2.1.17}$$

which is easier to solve. Indeed, (2.1.17) can be solved by matrix exponentiation, with the result

$$\begin{pmatrix} r^2 d\theta \\ r^2 sin\theta d\varphi \end{pmatrix} = \begin{pmatrix} \cos(cos\theta d\varphi) & -\sin(cos\theta d\varphi) \\ \sin(cos\theta d\varphi) & \cos(cos\theta d\varphi) \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} \tag{2.1.18}$$

where the constant differentials a and b – that can even be *fractals* – are constrained by the condition:

$$c^4 \equiv a^2 + b^2 \tag{2.1.19}$$

The two differentials in the left hand side of equation (2.1.18) have also the meaning of *elementary area components* on the surface of sphere. Indeed, from equations (2.1.1) and (2.1.6) we get:

$$\boldsymbol{r} \times d\boldsymbol{r} = (r^2 d\theta)(\hat{\boldsymbol{e}}_r \times \hat{\boldsymbol{e}}_\theta) + (r^2 sin\theta d\varphi)(\hat{\boldsymbol{e}}_r \times \hat{\boldsymbol{e}}_\varphi)$$

Therefore, the elementary area of the unit sphere is actually a vector in the plane (θ, φ), having as components the differentials from equation (2.1.18)

$$\boldsymbol{r} \times d\boldsymbol{r} = -(r^2 sin\theta d\varphi)\cdot\hat{\boldsymbol{e}}_\theta + (r^2 d\theta)\cdot\hat{\boldsymbol{e}}_\varphi \tag{2.1.20}$$

Thus our mention of the area law is, indeed, proper in three dimensions and has, among others, the form given in (2.1.13). Now, if it comes to introducing the dynamics, this can be done here in a classical way.

Notice that, according to the precepts of classical mechanics, as long as we have to do with *central* accelerations the last two of the equations (2.1.10) should not be affected by anything: it is only the first equation that acquires, for instance, a term in the right hand side. Consequently, the integral (2.1.13) persists even in this case, but the equation (2.1.14) gets, in its right hand side, an expression depending on the magnitude of the applied acceleration. So that instead of (2.1.14) we shall have:

$$d^2 r - r d\Omega^2 = f(\boldsymbol{r}) dt^2 \tag{2.1.21}$$

Here t is the time of the problem, as defined before, and $f(\boldsymbol{r})$ is the magnitude of the acceleration impressed according to the second principle of dynamics, up to a sign. This is a second-order purely differential equation for the magnitude of the position vector, to be solved once we know the magnitude of the acceleration. In order to go over to the new time here, we usually take notice that the equation (2.1.20), offer the ‘area constant’ of the motion by relation

$$r^2 \dot{\Omega} = \dot{a} \tag{2.1.22}$$

with an obvious notation for that area, as the rate of area swept by the position vector. Using this, (2.1.21) becomes the classical *Binet’s equation*:

$$u^2 \frac{d^2 u}{d\Omega^2} + u = \frac{1}{\dot{a}^2} f(\boldsymbol{r}), \quad u \cdot r = 1 \tag{2.1.23}$$

Now, that we have established the general mathematical frame of our discussion, it is time to appropriate it for an important case that, according to some ideas, *represents the very action of the inertia*.

2.2. Freedom According to the Idea of Central Forces

The classical Kepler problem, in its dynamical formulation, meant first and foremost a relief of spirit from the grips of classicism embodied in the identification of the action at distance with a force. First, it brought into theory the *concept of field*, and this concept is the one that liberated physics from the Newtonian burden of regularly evaluating the force with the aid of trajectory of motion. However, the idea of trajectory seemed to have been inescapable, insofar as, instead of force, the concept of field has brought in *the necessity of quantization*. This was necessary in order to allow for the theoretical description of the phenomenology at the microscopic scale of the world, where the light – in its electromagnetic stance, of course – would appear to be the defining phenomenon. However, even for the classical approach of natural philosophy, the concept of field means much more. Two of the essential cases related to the idea of centrality, with important impact in the construction of wave mechanics, will be presented in this section. They are essential, inasmuch as they constitute alternatives to quantization and electromagnetism, and important insofar as they offer the only possibility of properly exploiting the concept of interpretation.

The classical idea of field started with Poisson's equation, correlating the field with the existence of matter in space, as described by the Newtonian concept of density:

$$\nabla^2 V(\boldsymbol{r}) = 4\pi\rho(\boldsymbol{r}) \tag{2.2.1}$$

According to this view, the field offers forces in matter, by the gradient recipe:

$$\nabla V(\boldsymbol{r}) = \boldsymbol{f}(\boldsymbol{r}) \tag{2.2.2}$$

whose source is the concentration of matter as described by density. Apparently, this idea was firmly instated in the natural philosophy by Gauss as an application of his newly discovered theorem (Gauss, 1842). Now, if in a certain position from the universe the existing forces are central, which means that their action is only directed radially, then this equation must be written accordingly:

$$\nabla V(\boldsymbol{r}) = f(\boldsymbol{r})\frac{\boldsymbol{r}}{r} \tag{2.2.3}$$

where $f(\boldsymbol{r})$ is the magnitude of force which realizes the action describing the field, up to a sign. This equation can be written as

$$[\nabla V(\boldsymbol{r})]^2 = f^2(\boldsymbol{r}) \tag{2.2.4}$$

and allows us to discern a difference between the classical forces of dynamics and the forces acting on what we consider the *free particle*.

As well known, in dynamics the idea of free particle was axiomatically introduced by Newton (the second principle of dynamics), and thus one of the main problems of the classical dynamics was the description of inertia. Now, because by its very definition the classical free particle should not have accompaniment of any kind of matter *in sight* – and by this we understand the observation of any kind, even assisted by technology – its inertia was allotted by Newton to the absolute space. An allotment that – as by and large well known and publicized in physics, but not only there – proved to be quite inopportune from the point of view of the concept of field. This induced Ernst Mach into a revision of Newton's idea, with the consequence that a more realistic view was adopted, whereby the *matter out of sight* should be accepted as existent in the universe, and this is the one that controlls the inertia. It is the field theory then, that allows us to characterize the inertia as a force produced by field. This idea was instated in physics, in a radical way we might say, by a natural philosophy leading to the general relativity. However, we shall continue here the story along the classical lines.

In spherical coordinates at a certain point in space, we can write (2.2.4) as

$$[\nabla V(\boldsymbol{r})]^2 \equiv \left(\frac{\partial V}{\partial r}\right)^2 + \frac{1}{r^2}\left[\left(\frac{\partial V}{\partial \theta}\right)^2 + \frac{1}{sin^2\theta}\left(\frac{\partial V}{\partial \varphi}\right)^2\right] = f^2(\boldsymbol{r}) \qquad (2.2.5)$$

If the magnitude of the applied force depends only on the distance between particles, as in the case envisioned by Newton, then this equation is prone to a solution by separation of variables, as in the classical case of the solution of Hamilton-Jacobi equation. Indeed, in that case we can write (2.2.5) in the form

$$r^2\left[\left(\frac{\partial V}{\partial r}\right)^2 - f^2(r)\right] = -\left[\left(\frac{\partial V}{\partial \theta}\right)^2 + \frac{1}{sin^2\theta}\left(\frac{\partial V}{\partial \varphi}\right)^2\right] \qquad (2.2.6)$$

and assume tentatively a solution of the form

$$V(r,\theta,\varphi) = R(r) + F(\theta,\varphi) \qquad (2.2.7)$$

The equation (2.2.6) can have such a solution if, and only if

$$r^2\{[R'(r)]^2 - f^2(r)\} = -\left[\left(\frac{\partial F}{\partial \theta}\right)^2 + \frac{1}{sin^2\theta}\left(\frac{\partial F}{\partial \varphi}\right)^2\right] = -\beta^2 \qquad (2.2.8)$$

where β is a real constant. Thus we must have

$$R'(r) = \pm\sqrt{f^2(r) - \frac{\beta^2}{r^2}}, \quad \left(\frac{\partial F}{\partial \theta}\right)^2 + \frac{1}{sin^2\theta}\left(\frac{\partial F}{\partial \varphi}\right)^2 = \beta^2 \qquad (2.2.9)$$

The conclusion can be drawn here, that for*a field of central forces* having the magnitude going as the inverse distance between particles, the forces can have no radial component: the particles in such a field are 'radially free' with respect to each other. This property defines free particles, indeed, but *not* *from classical point of view*. From classical point of view no force whatsoever acts on a free particle, in any direction, while here, the particle are free only along the line joining them: the forces of magnitude

$$\pm\sqrt{[R'(r)]^2+\frac{\beta^2}{r^2}} \tag{2.2.10}$$

may have two transversal components, even in the case it has no radial component, which occurs if their magnitude is given by

$$f(r)=\pm\frac{\beta}{r} \tag{2.2.11}$$

Two historical incidents distinguish this kind of central forces among others, one of them connected with the concept of *interpretation*, the other with the concept of *wave*. In the first case, the forces were eliminated by Maxwell from the description of classical ideal gas, on the ground that, being long-distance forces, they cannot act in an ensemble of material points representing an ideal gas in thermodynamical equilibrium [(Maxwell, 1965), Volume II, p. 422]. Among other things, this elimination led directly to the Planck's theory of quanta, based on which we tried recently to rehabilitate the forces (2.2.11), according to the modern idea of sufficiency in theoretical statistics (Mazilu *et al*., 2020). The second incident concerns the idea of waves in their electromagnetic instance. Even from the times of Hooke, light was the epitome of *lateral action* without anapparent accompanying radial one. On the occasion of arising of special relativity the scientists were forced to recognize that there are two kinds of electromagnetic waves propagating with the speed of light [(Langevin, 1905); see also (Poincaré, 1906)]: the 'velocity waves' and the 'acceleration waves'. These are distinguished from one another by the fact that they fade away differently: while the velocity waves fade away with the inverse of the square of distance, like the Newtonian forces, the acceleration waves fade away with the inverse of that distance. In other words, the acceleration waves reach further away than the velocity waves, so that Dennis Sciama was tempted to put the inertia in their charge (Sciama, 1969), a fact that seems just natural under the auspices of general relativity. In the spirit of what we have just presented thus far in this work, it is perhaps best to reproduce two of the Langevin's conclusions in his own words. Quoting, therefore:

> 1° The electromagnetic perturbation produced in the medium by an *electrified particle* (*our Italics here, a/n*) in motion is composed of two parts that propagate with the speed of light, starting from the emission center.
>
> The first part, or the *velocity wave*, which *exists only in the case of rectilinear and uniform motion* (*our Italics here, a/n*), depends only on

the velocity of mobile; it contributes in developing around this one a wake whose energy varies with the velocity, which therefore contains the kinetic energy related to the electrified center, and which accompanies this one in its displacement, modifying itself if the motion is accelerated;

2° This modification is produced through the intermediary of second part of the perturbation, the *acceleration wave*, having at *any distance* from the emission point *the properties of transversality and equality of the electric and magnetic energies* (*our Italics here, a/n*), which correspond to the free radiation.

This *acceleration wave* transports at a great distance, *where the velocity wave becomes negligible* (*our Italics here, a/n*), a finite energy proportional to the square of acceleration and increasing indefinitely with the velocity when this one approaches that of light. The polarisation properties of this wave are particularly simple when the velocity is small.

The velocity wave does not transport any energy at great distances; the *energy of the corresponding wake* only follows the center in its displacement;

…

The preceding considerations seem to cast some light on the intimate mechanism of the phenomena of inertia and radiation. [(Langevin, 1905); *our translation, original Italics, except as indicated*].

These old classical conclusions became routine in the modern electrodynamics [see (Jackson, 1998), Chapter 14, Eqs. (14.13–14)]. Our concern is that, while they can simply be supported by dynamical considerations expressed in the field theoretical frame work, as we have shown above, they can be also expressed in the language of waves. This, in our opinion, is an indication of the compelling necessity of going along with a *reverse interpretation*, as we would like to call the process, whereby from a 'detached' element, like a material point, we need to pass to a continuum having the properties of the other detached elements, missing in actuality but virtually existent. This, we believe, is the key to *a physical theory of the memory*, and it is by no means specific to brain, but to matter of the universe at large: *the inertia*.

For once, let us now discuss the transversal components of the force describing the classical field, as given by the second of the equations (2.2.10). It can be written as well in a form obviously separable again:

$$sin^2\theta\left[\left(\frac{\partial F}{\partial\theta}\right)^2-\beta^2\right]=-\left(\frac{\partial F}{\partial\varphi}\right)^2=-\gamma^2 \qquad (2.2.12)$$

where γ is a real constant. Repeating the procedure of separation, we put

$$F(\theta,\varphi)=\Theta(\theta)+\Phi(\varphi) \qquad (2.2.13)$$

so that (2.2.12) splits into two ordinary nonlinear differential equations:

$$[\Theta'(\theta)]^2 = \beta^2 - \frac{\gamma^2}{sin^2\theta}, \quad [\Phi'(\varphi)]^2 = \gamma^2 \tag{2.2.14}$$

Thus, we have the general result that the force of magnitude *f*(*r*), describing a classical field, can be written in the form

$$\boldsymbol{f}(\boldsymbol{r}) = \frac{1}{r}\Big\{\pm\sqrt{[rf(r)]^2 - \beta^2}\cdot\hat{\boldsymbol{e}}_r + \frac{1}{sin\theta}\Big[\pm\sqrt{\beta^2 sin^2\theta - \gamma^2}\cdot\hat{\boldsymbol{e}}_\theta \pm \gamma\cdot\hat{\boldsymbol{e}}_\varphi\Big]\Big\} \tag{2.2.15}$$

where the different combinations of signs in the curly brackets are to be conveniently used in case they are needed. For the case from equation (2.2.11) this force can be written as

$$\boldsymbol{f}(\boldsymbol{r}) = \frac{1}{rsin\theta}\Big[\pm\sqrt{\beta^2 sin^2\theta - \gamma^2}\cdot\hat{\boldsymbol{e}}_\theta \pm \gamma\cdot\hat{\boldsymbol{e}}_\varphi\Big] \tag{2.2.16}$$

with the same observation. Obviously, the reality of the components of force requires definite conditions on the ranges of coordinates, but just for the sake of argument we will ignore these for the moment, along with the ambiguity of sign, assuming that they are conveniently satisfied.

In fact, in order to eliminate any confusion that may appear due to the fact that the previous solution of the equation (2.2.5) is incidental, we shall assume a general situation of a field generated by a material point, manifest by forces in the space surronding it, having no radial component. Therefore, these forces are vectors like those from equation (2.2.16), only with some generic components:

$$\boldsymbol{f}(\boldsymbol{r}) = f^\theta\cdot\hat{\boldsymbol{e}}_\theta + f^\varphi\cdot\hat{\boldsymbol{e}}_\varphi \tag{2.2.17}$$

The components of this vector can be any real functions of position in space, so that we can conduct the mathematics with due generality. The only fact that may appear as incidental here is the starting assumption: *no radial component of such forces*.

2.3. The Radial Component of the Forces

We can think of an ensemble of classical material points in equilibrium in any direction at any distance: this is an ensemble of material points endowed with gravitational mass and charges, electric and magnetic (see II, §§2 & 3). As we have shown, this is a *fictitious* ensemble, inasmuch as any *real* ensemble of material particles endowed with masses and charges cannot be in static equilibrium. However, as we also have shown (*loc. cit.*), the kind of nonequilibrium described through Newtonian forces generated by mass and charges is a feature depending on the space scale where we consider the nonequilibrium. For instance, at the transfinite scale of space, the gravitation dominates and therefore the forces are dominantly attractive, while at an infrafinite or finite scale the charges dominate, and the forces are dominantly repulsive. Of course, in these cases we think of interpretation in terms of an ensemble of *identical particles*. By the very same token, we are allowed to think of an ensemble in equilibrium under Newtonian forces, defined by a condition independent of space scale: every particle of the ensemble is stationary, inasmuch as it is in equilibrium with any other identical particle, located at any distance in any direction. The condition of this equilibrium generates an algebraic equation giving us the possibility of a geometrization of the physical quantities describing the matter (*loc. cit.*). The resulting geometry is a Cayleyan geometry.

We shall call the classical material point of such a fictitious ensemble a *Hertz material particle*, in view of his inventor, who also indicated the scale character of the eventual geometry describing such an ensemble and its elements. This is a case apparently never taken into consideration by the natural philosophy of any persuasion – forgotten at its very birth, if we may say so – and which was aroused, the first and only time, by Heinrich Hertz in his *Principles of Mechanics* (Hertz, 2003). Probably, even Hertz himself did not know what to do with the concept – the course of his beautiful work follows only the mathematical line of presenting the principles of mechanics, 'in a new form', as he says – but the truth is that only the wave and quantum mechanics unveiled the true gnoseological capabilities of this concept, all converging mostly in the definition of interpretation by Charles Galton Darwin. This turned out to be the fundamental concept of physics, and around it, explicitly or implicitly, the whole modern physics has been built (Mazilu *et al.*, 2019). The best illustrative example is the discussion around the cosmological problem started by Einstein in 1917 (Mazilu *et al.*, 2020). It revealed the important position of the Einsteinian point of viewin the natural philosophy, and the clear difference between this and the old classical Newtonian point of view. It also revealed that the two points of view can be 'reconciled', if we may say so, into a general, apparently more realistic, natural philosophy, whereby the wave mechanics plays an essential part. As here we just have to pinpoint that part, it seems better to start from the very Hertz's concept.

We reproduce and discuss, for now, only the necessary original definitions and commentaries [(Hertz, 2003), pp. 45–46)], keeping in store our understanding, to be revealed gradually and, of course, specifically, as we go along with our work. Therefore, quoting:

> **Definition 1.** A *material particle is a characteristic* by which we associate without ambiguity a given point in space at a given time with a given point in space at any other time.

> Every material particle is *invariable and indestructible*. The points in space which are denoted at two different times by the same material particle, coincide when the times coincide. Rightly understood the definition implies this.
>
> **Definition 2.** The number of material particles *in any space*, compared with the number of material particles *in some chosen space at a fixed time*, is called *mass* contained in the first space.
>
> We may and shall consider the number of material particles in the space chosen for comparison to be infinitely great. The *mass of the separate material particles* will therefore, by the definition, be *infinitely small*. The mass in any given space may therefore have any rational or irrational value.
>
> **Definition 3.** A *finite* or *infinitely small mass*, conceived as being *contained in an infinitely small space*, is called a *material point*.
>
> A material point therefore consists of any number of material particles *connected with each other*. This number is always to be infinitely great: this we attain by supposing the material particles to be of a *higher order of infinitesimals* than those material points which are regarded as being of infinitely small mass. The masses of material points, and especially the masses of infinitely small material points, may therefore bear to one another any rational or irrational ratio (*our emphasis, n/a*).

The trend of progressing of his *Mechanics* does not seem to indicate that Hertz followed a program as outlined in this list of definitions, at least not from the points of view later revealed in physics. It seems just normal: the reasons listed by Hertz in his *Preface* to the treatise show that he followed mainly the soft spots of the concept of force at that time. This is perhaps the reason that the treatise does not play today, as it never did,in fact, the foundational part it deserves in our physical knowledge. However, an early analysis by Henri Poincaré suggests that Hertz's work has to be taken more seriously even as it is, for even as such it touches fundamental issues of the human knowledge. Quoting:

> I insisted on this discussion longer than Hertz himself; I meant to show though that Hertz didn't simply look for quarrel with Galilei and Newton; we must agree to the conclusion that in the framework of the classical system *it is impossible to give a satisfactory idea for force and mass* [(Poincaré, 1897); *our translation, original emphasis*].

The exquisite analysis of the great scholar, and everyone else's ever interested in the *Mechanics* of Hertz, for that matter, does not appear to take due notice of the concepts involved in the excerpt above. Fact is that the excerpt, which apparently is referring only to mass, touches actually, even though partially and perhaps only implicitly, both of the two fundamental ideas mentioned by Poincaré, and with them the objective reasons of the subsequent general relativity and wave mechanics. In this respect, two things are worth noticing right away, bearing directly on our subject-matter here.

The second point of Hertz's definitions, which is very important for us, is an implicit consideration of the space scale. Namely, in his material particles one can easily recognize the classical material points: *positions endowed with physical properties*. In view of our Newtonian definition of the three physical properties, such material points cannot be but fictitious. However, a *material particle* to Hertz endows, according to Newtonian view, a position in space not only with an identity, a task for which the definition is mainly intended, but with an indestructible material 'anchor', as it were, when that position is in the matter. Indeed, it is not too hard to see that, besides being indicator for a point in space, such a material particle is, incidentally, the most convenient *point of application of a force describing the field*. And thus itcan also support the third principle of dynamics, inasmuch as it is conceived as dimensionless and, more than that, "invariable and indestructible". This fact is crucial for building an interpretation.

Now, there is a critical difference between a position in space and a position in matter: in the first kind of position the particle possesses *motion*, in the last it does not. However, let us recall that, in modeling the reality around us we have only the possibility to work with material points in the acceptance of Hertz. Indeed, we are aware that a star, for instance, is actually an extended body, and only from a distance we see it as a point. Therefore we have to accept that a material point is itself a complicated structure, and this fact is duly noted in Hertz's formalism: *the material points are made of material particles*! This very definition liberates our spirit from the necessity of removing the forces from the stage of a physical theory, as the general relativity claimed sometimes to have been doing. Indeed, a material particle can support an acting force on it, and this acting force is always present with its defining reacting force. In a word, in the structure of a material point, a material particle is as 'free' as it gets, given the environment. As a matter of fact, this was entirely Newton's initial philosophy!

There is a subtle point here that unfortunately has not been exploited along the time, because of the prevailing concept of vector attached to force. In order to reveal it, let us notice that Hertz's definitions *implicitly* show that there is a real difference between *motion* and *displacement*. This difference enters the definition of *material point*: forces may act on a material point only through its constituent material particles. These material particles can only be *displaced* by forces. However the *motion* may not be a direct consequence of the force as in the Newtonian axiomatics. As a matter of fact strange situations may appear where the point of application of a force acting on a material point is outside of *anyone* of the material particles from the constitution of that material point. As long as we maintain the geometrical image of vector for a force, like Hertz did, we may not have too much of a choice in overcoming this difficulty but to define further notions which are "concealed" [(Hertz, 2003), pp. 223–225]. We believe that the real lesson to be learned here is that we have to speak, generally, of a *material point* in the sense of Hertz when describing a motion we happen to observe, and of *material particles* in the sense of Hertz when in need of properly describing the action of forces that might go along with this motion. However, when it comes to describing the force as an *effect of motion*, we need to pay close attention, because some statistics may come into play, as dictated by the scale where we contemplate the things. After all, a material point is, first and foremost, and *ensemble of material particles*! And when it comes to ensemble, the statistics is most appropriate method to use. An example in point follows immediately.

Assume an ensemble of Hertz material particles *in equilibrium*, and settle to discuss *one of them* at random, from a 'central' point of view. Then there is no nonzero component of force along any line joining it with any other particle, so that any other particle has a force on it due to the central one, given by equation (2.2.17). This force can vary from particle to particle, so that its variation acquires a radial component due to the variation of the frame ($\hat{e}_\theta$, $\hat{e}_\varphi$):

$$d\boldsymbol{f}(\boldsymbol{r}) = -(f^\theta d\theta + f^\varphi sin\theta d\varphi)\cdot\hat{\boldsymbol{e}}_r + (df^\theta - f^\varphi cos\theta d\varphi)\cdot\hat{\boldsymbol{e}}_\theta + (df^\varphi + f^\theta cos\theta d\varphi)\cdot\hat{\boldsymbol{e}}_\varphi \tag{2.3.1}$$

Here we used the Frenet-Serret equation (2.1.5) for the variation of the reference frame ($\hat{e}_\theta$, $\hat{e}_\varphi$). There is a subtle formal point here, that decides the statistical character of this expression: if it describes a free particle – the central one – this description is done in connection with the other particles of the ensemble in static equilibrium with that particle. Therefore, the equation (2.3.1) describes an ensemble of *biparticles*, involving the distance between two particles in a reference frame deciding the directions in space.

The problem is to construct a reference frame that does not depend on the particle, but on the biparticle, in view of the action of force. Such an Euclidean reference frame is given by the matrices:

$$\hat{\boldsymbol{e}}_1 = \begin{pmatrix} 2(x^2/r^2) - 1 \\ 2(xy/r^2) \\ 2(xz/r^2) \end{pmatrix}; \hat{\boldsymbol{e}}_2 = \begin{pmatrix} 2(xy/r^2) \\ 2(y^2/r^2) - 1 \\ 2(yz/r^2) \end{pmatrix}; \quad \hat{\boldsymbol{e}}_3 = \begin{pmatrix} 2(xz/r^2) \\ 2(yz/r^2) \\ 2(z^2/r^2) - 1 \end{pmatrix} \tag{2.3.2}$$

One can easily verify the *orthonormality* of this frame, and the important relations that define a position vector $\boldsymbol{r}$, regardless of its origin:

$$\boldsymbol{r}\cdot\hat{\boldsymbol{e}}_1 = x,\quad \boldsymbol{r}\cdot\hat{\boldsymbol{e}}_2 = y,\quad \boldsymbol{r}\cdot\hat{\boldsymbol{e}}_3 = z \tag{2.3.3}$$

Thus, even though the quantities x, y, z may be taken completely *arbitrarily* – provided they are finite, of course – they *can be interpreted as Euclidean coordinates*. We have, for instance, the case of Shpilker coordinates (see II, §9), well suited for the de Broglie's physics of light ray (see I, §2.2). This conclusion has a remarkable connotation: if any Riemannian space – in fact, any space – can be discussed in terms of Euclidean reference frames (Cartan, 1930, 1931), the equation (2.3.2) provides a suitable reference frame, necessary for that description. However, an important precaution must be exercised, for usually in the case of the Riemannian geometry describing a certain space – such as in the case of matter – such a description must be done not in terms of *curvature*, but in terms of *torsion*. We shal come to this topic in concluding this instalment of our work.

For now, going along with the geometrical line of reasoning, the Frenet-Serret equations for the frame (2.3.2) can be obtained by direct calculations. As the procedure involves some calculational features that will be also necessary later on, we will describe it briefly here. Notice first that the frame definition from equation (2.3.2) involves only coordinates on the unit sphere, relative to central particle. These are the components of the unit vector $\hat{\boldsymbol{e}}_r$ from equation (2.1.2). We denote these coordinates with ξ, η, ζ:

$$\xi = \frac{x}{r},\quad \eta = \frac{y}{r},\quad \zeta = \frac{z}{r} \tag{2.3.4}$$

Their differentials are to be calculated directly from their definition, and are given by the formulas

$$\frac{dx}{r} = d\xi + \xi\frac{dr}{r},\quad \frac{dy}{r} = d\eta + \eta\frac{dr}{r},\quad \frac{dz}{r} = d\zeta + \zeta\frac{dr}{r} \tag{2.3.5}$$

Now starting from the definition of the Frenet-Serret matrix entries:

$$\Omega_{jk} = \hat{\boldsymbol{e}}_k\cdot d\hat{\boldsymbol{e}}_j \tag{2.3.6}$$

we find a skew symmetric matrix having the elements

$$\begin{gathered}\Omega_{12} \equiv \hat{\boldsymbol{e}}_2\cdot d\hat{\boldsymbol{e}}_1 = 2\frac{xdy - ydx}{r^2}, \Omega_{23} \equiv \hat{\boldsymbol{e}}_3\cdot d\hat{\boldsymbol{e}}_2 = 2\frac{zdy - ydz}{r^2},\\ \Omega_{31} \equiv \hat{\boldsymbol{e}}_1\cdot d\hat{\boldsymbol{e}}_3 = 2\frac{xdz - zdx}{r^2}\end{gathered} \tag{2.3.7}$$

In other words, the entries of the Frenet-Serret matrix are components of the differential vector representing the *elementary spherical angle centered on a particle*. It should be worth our while writing this infinitesimal vector in terms of the spherical coordinates. We have, using the definition of the spherical coordinates,

$$\begin{aligned} \Omega_{12} &= 2sin^2\theta d\varphi \\ \Omega_{23} &= 2sin\varphi d\theta + sin2\theta cos\varphi d\varphi \\ \Omega_{31} &= -2cos\varphi d\theta + sin2\theta sin\varphi d\varphi \end{aligned} \tag{2.3.8}$$

Now, we have to calculate the differential of a vector in this reference frame, just as we have done before for the reference frame related to spherical coordinates.

Thus, let us consider the vector force, whose components with respect to the reference frame (2.3.2) are taken as contravariant components:

$$\boldsymbol{f}(\boldsymbol{r}) = f^k \cdot \hat{\boldsymbol{e}}_k \tag{2.3.9}$$

The differential of this vector is

$$d\boldsymbol{f}(\boldsymbol{r}) = df^k \cdot \hat{\boldsymbol{e}}_k + f^k \cdot d\hat{\boldsymbol{e}}_k = (df^k + f^j \Omega_j^k) \cdot \hat{\boldsymbol{e}}_k \tag{2.3.10}$$

where we have used the Frenet-Serret equations for the frame, and arranged things so that the superior index to be used in the summation convention is the index of the vector which in formula (2.3.6) does *not* appear under the differential operation. Thus, the components of the differential of force can be written as

$$Df^k = df^k + f^j \Omega_j^k \tag{2.3.11}$$

The same considerations for the components of the position vector, lead to the components

$$\omega^k = -dx^k + x^k \frac{dr}{r}, \quad d\boldsymbol{r} \equiv \omega^k \hat{\boldsymbol{e}}_k \tag{2.3.12}$$

Thus, an elementary work of the force can be written as

$$dW \equiv \boldsymbol{f} \cdot d\boldsymbol{r} \equiv \sum f^k \omega^k \tag{2.3.13}$$

and this is not an exact differential. It is such an exact differential if

$$d \wedge dW \equiv \sum df^k \wedge \omega^k = 0 \tag{2.3.14}$$

which, in view of (2.3.12), comes down to

$$\sum df^k \wedge dx^k = 0, \quad \left(\sum x^k df^k\right) \wedge \frac{dr}{r} = 0 \tag{2.3.15}$$

According to Cartan's Lemma, these conditions are equivalent to

$$df^k = \Lambda_m^k dx^m, \quad \sum x^k df^k = \Lambda \frac{dr}{r} \tag{2.3.16}$$

where $\boldsymbol{\Lambda}$ is a symmetric matrix and Λ is a scalar, both of them *conveniently chosen*, and therefore can assume a physical origin. Now, if we describe the statistics of forces by the virial of Clausius, *i.e*. the quantity $\boldsymbol{f}\cdot\boldsymbol{r}$, defined in any position of the ensemble (Clausius, 1870), then this quantity should be a constant, and therefore its differential should be zero. Thus we need to have

$$\sum f^k dx^k + \sum x^k df^k = 0 \tag{2.3.17}$$

or, using (2.3.16)

$$dW + \Lambda\frac{dr}{r} = 0 \tag{2.3.18}$$

which shows that, if dW is an exact differential it is the differential of the logarithm of distance. In the case of transversal forces describing the field of a particle, their variation over the ensemble of equilibrium, as described from the point of view of an arbitrary particle, is given by the formula (2.3.1), so that we have

$$\boldsymbol{r}\cdot\boldsymbol{df} = -r\left(f^\theta d\theta + f^\varphi sin\theta d\varphi\right) = \beta\frac{dr}{r} \tag{2.3.19}$$

where β is a constant, so that

$$\int\left(f^\theta d\theta + f^\varphi sin\theta d\varphi\right) = \frac{\beta}{r} \tag{2.3.20}$$

So, the ‘celestial mean’ of a force along a path that goes ‘in depth’ from a center point, is a *logarithmic force*: in other words, if from a central point of view we have a path of forces deriving from a logarithmic potential then the particles are radially free. Such a motion is, for instance the motion of a charge in the field of a magnetic pole, which, described *dynamically*, is a spiral on a conical surface (Poincaré, 1896).

However, notice that a geodesic on a conical surface may very well represent a classical light ray in the sense of de Broglie, and that the equation (2.3.20) may represent a mean of the transversal force on an ‘arc of geodesic’, as it were. These very facts, combined with the one that the quantity in question is going with the inverse distance along the ray, recommend the condition (2.3.20) as a de Broglie condition of ‘approaching the particle at constant time’ [see I,§2.2, equation (2.9)]. The noteworthy part in this conjecture is that such a condition should be universal, and carry the light’s property of scale transition, for which we need to find the right expression.

3. The Matter Content of the Celestial Vault

There is nothing, indeed, illustrating either the idea of freedom of particles, or that of scale transition, in a central point of view, better than the case of light. The light was, from the very beginning, interpreted in terms of rays of fictitious free particles, these even having no special name. To Newton these particle were acted upon by ponderous matter with forces normal to the surface of the matter [(Newton, 1952), pp. 79ff]. To Hooke the motion in orb was periodical ‘short motion’ [(Hooke, 1665), pp. 55–56]. However, it is hard to tell what would be the geometrical expression of this idea of freedom in the case of *observed* matter. One natural suggestion on such a tellingis that this geometry must generalize the geometry of that motion that generated the Newtonian system of mathematical philosophy: first, the free motion, and then the Kepler motion. We believe that this is the case, so much the more as this belief is supported by an idea of scale transition. Let us elaborate on these observations.

In order to illustrate the case, we will describe now the *plane anharmonic curves*, a class of plane curves that comprise the conics, together with spirals and a series of other shapes exhibited either implicitly, like the Kepler motion, or in the direct observations of the celestial matter. We follow here closely the work Nicolae Mihăileanu, which, as its title shows, completes, indeed, any course lectures of differential geometry with pertinent applications (Mihăileanu, 1972). This author presents the plane anharmonic curves as curves having a homographic relation between the slope of their tangent in a point and the slope of the secant in that point that passes through a fixed point in plane – the pole of the curve. This is a truly scale transcendent relation.

3.1. Geometrical Description of the Shapes of Matter

Let (u, v) be the plane coordinates in a certain reference frame. According to the definition, if $v(u)$ is the function involved in describing the anharmonic curve and $v'(u)$ is the slope of the tangent in a point of this curve, then the equation of this curve is given by

$$v'(u) = \frac{av + bu}{cv + du} \tag{3.1.1}$$

where a, b, c, d are some coefficients supposed here to be real. The numerical characteristics of the 2×2 matrix:

$$\boldsymbol{a} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{3.1.2}$$

viz. the fixed points of its homographic action and the eigenvalues of its linear action in two dimensions decide the geometrical shape of the curve described by equation (3.1.1). This fact is quite obvious from the direct integration of equation (3.1.1). Indeed, we can put it in the form:

$$-\frac{1}{v}\frac{dv}{d\xi} = \frac{a\xi + b}{c\xi(\xi - \xi_1)(\xi - \xi_2)}, \quad \xi = \frac{v}{u} \tag{3.1.3}$$

where $\xi_{1,2}$ are the fixed points of the matrix (3.1.2), *i.e*. the roots of the quadratic equation:

$$c\xi^2 + (d - a)\xi - b = 0$$

Geometrically, these numbers are the slope of the asymptotes of curve described by equation (3.1.1).

Assuming that the matrix (3.1.2) has distinct eigenvalues, λ_1 and λ_2 say, the result of integration of the equation (3.1.3) is

$$\frac{|v - \xi_2 u|^{\lambda_2}}{|v - \xi_1 u|^{\lambda_1}} = K \tag{3.1.4}$$

because the fixed points are distinct along with the eigenvalues, and of the same algebraical nature; here K is an integration constant. Accordingly, if the eigenvalues are real, the curve thus described by the matrix (3.1.2), *i.e. via* the action (3.1.1), is a real parabola of degree λ_1/λ_2. On the other hand, if the two eigenvalues are complex, so that we can write

$$\lambda_{1,2} = \alpha \pm i\beta$$

the anharmonic curve is a logarithmic spiral that can be considered as the projection of a *normal* logarithmic spiral on some differently oriented plane:

$$\rho = Ke^{-m\psi} \tag{3.1.5}$$

where K is, again, a real constant, and we used the notations

$$\beta\rho^2 = (\beta u)^2 + (v - \alpha u)^2; \quad tan\psi = \frac{\beta u}{v - \alpha u}, \quad m = \frac{d + \alpha c}{\beta c}$$

There is one more case, of identical eigenvalues, and therefore identical fixed points, where the curve described by matrix ***a*** has a unique asymptotic direction. Denoting λ, respectively μ, the two numerical characteristics of the matrix, the equation (3.1.3) can be written in the form

$$-\frac{1}{v}\frac{dv}{d\xi} = \frac{a\xi + b}{c\xi(\xi - \mu)^2}$$

which can be integrated directly, with the result

$$ln|v - \mu u| + \frac{\lambda}{c}\frac{|u|}{|v - \mu u|} = K \tag{3.1.6}$$

where K is the integration constant, assumed real. We have here too a projection, but of an exponential curve.

Summing up, the conics used by Kepler in building his synthesis of planetary motion are not the only curves present in the sky. We have also the spirals, also present in the sky in the form of galaxies, but unknown to Kepler, from which the conics are obtained by making m=0 in equation (3.1.5).

Moreover, we have some other shapes, given by the equation (3.1.6), that do not seem to have celestial correspondents. However, the equation (3.1.6) is the only one that may correctly describe the fall toward a center of force, and therefore it could faithfully describe a motion of matter proper under the action of gravitation: it is the motion involved in *accretion* of matter. Indeed, the equation (3.1.6) can be read in the following way: at great distances from the attraction center, a certain body falls directly toward the center following the asymptote. It starts gradually departing from the asymptote, so that in close range it has a 'roundabout' motion approaching asymptotically the center of force. According to this image the free fall is a reality only at large distances from the center of force. Thus, we may say that the Galilei kinematics, for instance, is a kind of asymptotic limit of the real kinematics, represented by an exponential curve of the type described by equation (3.1.6). As we shall see, this allows a precise scaling of the radial distance. Until then, it is time to show, in connection with the previous results, why do we insist on the light properties in the description of a universe.

3.2. Clasical Results: Necessity of Time

In obtaining the previous results a certain feature of the reasoning is manifest and needs to be analyzed in depth: none of the curves thus obtained is described parametrically, all of them are gotten implicitly, starting from differential forms by integration.The integration is here understood as an operation inverse to the operation of differentiation. In order to get these curves in a parametric form, we need to introduce a *continuity parameter*, therefore, in physical terms, to introduce the time, and this requires an equation by the means of which the time can be quantified. Classically this was the second of the Kepler's laws.

In order to settle the ideas, let us assume explicitly what up to this point was tacitly assumed, namely that the elements of the matrix $\boldsymbol{a}$ from the equation (3.1.2) are constants: the matrix does not vary. We can then replace the equation (3.1.1) by a system of two differential homogeneous equations with respect to a continuity parameter, tentatively denoted t in view of the fact that it can, in the last resort, represent the *time*. Let us therefore write (3.1.1) as:

$$du = (Cv + Du)(\nu dt), \quad dv = (Av + Bu)(\nu dt) \qquad (3.2.1)$$

where, this time, capitals are used for the coefficients in order to avoid confusion with the customary symbol 'd' of the operation of differentiation, and ν is an arbitrary parameter of convenience, that may be even variable. The equation (3.2.1) makes obvious what we have anounced before, and the equation (3.1.1) contains only in a veiled fashion: as the equation (3.2.1) is a linear transformation between the finite parameters and their differentials, *the anharmonic curves are the expression of a transcendence between infrafinite* *and finite space scales*. This observation may not be, technically speaking, too much, but it raises an issue of principle. Let us, therefore, digress a little of this issue.

We insist, once again, on the idea that the light is the only physical signal transcending the space scales. This is, after all, the way the physics took its modern shape: the signals apparently coming from *transfinite* spaces are only perceived within *finite* spaces, and the perception is physically explained within *infrafinite* spaces. The legitimacy of this explanation is secured by the fact that the cosmic background radiation, for instance, has definitely a Planck spectrum (Fixsen *et al*., 1996), and therefore obeys the Wien displacement law, which is an expression of the scale invariance in the physics of light (Mazilu, 2010). Now, as we have seen here, the spiral structure revealed by the light we perceive from the space at large, can be geometrically explained – see equation (3.1.5) – as an anharmonic family of trajectories of some particles, usually identified with stars. However, this identification needs to be taken *cum grano salis*, as it were, inasmuch as there are discrepancies between the geometry revealed by observations, and any possibility whatsoever of physical explanation of the cosmic structures: missing mass, dark matter and the like. Nonetheless, if there should be some truth in such an identification, it can only be revealed if one asks the question: *is the observed celestial structure real, or it is only the appearance due to the properties of transcendence of light*?! Everything in the explanation of the physical universe corresponding to the finite space scale decided by the existence of Earth, is pending on the answer to this question. It should be, therefore, the hope allowed, that physics of brain will open the gate for some answer to this question.

Coming back to our mathematics here, the equation (3.2.1) can be written in the obvious matrix form:

$$|\dot{u}\rangle = \upsilon \cdot \boldsymbol{a} \cdot |u\rangle; \quad |u\rangle = \begin{pmatrix} u \\ v \end{pmatrix} \tag{3.2.2}$$

where a dot over a symbol means time derivative, as usual. If the parameter ν is also constant or, if variable, can be incorporated into the definition of either time or coordinates by a suitable scaling, we can also calculate the second derivatives of coordinates with respect to time. These are necessary in order to express the fact that the second of the Kepler's laws is respected in the form: the rate of area variationof the position with respect to the pole of motion is constant. Using the equation (3.2.2) this comes down to equation

$$|\ddot{u}\rangle = (\upsilon \cdot \boldsymbol{a})^2 \cdot |u\rangle \equiv \boldsymbol{b} \cdot |u\rangle \tag{3.2.3}$$

Now, the condition that the second of the Kepler laws be satisfied, has a precise formulation:

$$v\ddot{u} - u\ddot{v} = 0 \tag{3.2.4}$$

which, using the equation (3.2.3), becomes

$$b_{12}v^2 + (b_{11} - b_{22})uv - b_{21}u^2 = 0$$

independently of u and v. This tells us that the matrix $\boldsymbol{b}$ must be the 2×2 identity matrix, up to an arbitrary factor. By choosing, if allowed, the factor μ appropriately, we can determine the matrix $\boldsymbol{a}$ such that its square is the identity matrix up to a sign:

$$\boldsymbol{a}^2 = \mathbf{1}$$

According to Hamilton-Cayley theorem, this means that **a** must have a null trace, and unit determinant up to sign. In suggestive notations for the entries of **a**, we can write it as:

$$a = \begin{pmatrix} -a_{12} & -a_{22} \\ a_{11} & a_{12} \end{pmatrix} \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}$$

Thus the equation (3.2.2) can be written as an exact differential

$$(a_{11}u + a_{12}v)du + (a_{12}u + a_{22}v)dv = 0 \qquad (3.2.5)$$

so that it can be integrated by inverse differentiation:

$$a_{11}u^2 + 2a_{12}uv + a_{22}v^2 = K \qquad (3.2.6)$$

Here K is a new constant, introduced by integration procedure. In this case, the motion described by the differential equation (3.2.5) is a Hamiltonian motion, for which the quadratic form (3.2.6) is the very Hamiltonian, and also provides a conservation law. The coordinates u and v have now a precise meaning: they are the coordinates of motion with respect to the center of orbit described by equation (3.2.6). We thus have the important result that in order to have the area law satisfied, in a 'universal manner' as it were, whereby *the area constant is not specified*, the celestial orbits *must be conics*. Inasmuch as the second of the Kepler laws may not be satisfied for them, we are entitled to say that the *anharmonic motions generalize the classical planetary Kepler motions*. Consequently they can be taken as an essential step in construction of a necessary modern synthesis that generalizes the classical Keplerian one, and can serve the modern physics the way the Keplerian synthesis served the classical physics.

4. By Way of Conclusions: Cartan's Geometrical Philosophy

Let us repeat again: we aim to accomplish the idea that the living brain should be physically modeled as a universe. The analogy goes, if properly caried, to details: the constitutive unit of the brain universe matter – the neuron – should handle the charges just like the constitutive unit of the matter of the common universe – the planetary model – namely in the manner the rays of light handle their constitutive particles: photons, luxons, etc. The problem is: what are these constitutive particles for the case of matter? The answer was presented here by Hertz's material particles: fictitious classical material points, endowed with physical properties of gravitational mass, electric charges and magnetic charges, allowing us to interpret the matter by ensembles in static equilibrium due to the forces commonly associated as vectors with these physical features.

The matter of brain is like the matter of usual universe, or of any universe actually. However, we owe the reader an explanation, because the question can arise: why should we insist in describing the matter of celestial vault? What is the point of analogy it serves?! From physical point of view, we should not forget the experience by any means. In the case of brain this experience is represented mainly by a phenomenology of neurological extraction, and when it cames to experiments, these are only experiments of nondestructive nature, involving the electric and magnetic properties of the brain. So, we can 'see' the brain exactly as we see the universe at large: by some 'windows' opened for us through the very structure of the universe. And in this last case, the windows are the celestial matter formations just described above. The analogy means that here the external observation of brain should be physically modelled just as we model the light coming from celestial objects, and the devices of this observation should function accordingly. The problem is why Euclidean reference frames? Knowing that these are, geometrically speaking, only extremely special reference frames, can they be in any way connected to the essence of any physical problem at all?

The concept of interpretation raises quite a significant concern, in hindsight even critical, which tends to show up especially if we forget about that objective connotation that Schrödinger assigned to his wave function, the one involving the idea of a 'charge cloud' (Schrödinger, 1920). Incidentally, we have to recognize, though, that such an interpretation is, in fact, the largest level of acceptance ever possible, inasmuch as it includes, as only a particular case, that experimental level invoked by Darwin in defining the concept of interpretation (see I, §2.3), a fact that shall be obvious as we go along with our work. The concern we are expressing by the virtual questions raised above, is conspicuous, if we may say so, and comes with that notable dichotomy regarding the problem of space, which we have only mentioned above, but, nevertheless, needs to be pinpointed as such. That dichotomy actually confounds the whole human knowledge of all times – no matter if natural philosophical, purely philosophical or simply technical in general – being, in fact, unrecognized as such even today. It is the difference between what is *philosophically* accepted as 'the ordinary space', and what is *scientifically* accepted as 'the coordinate space'. Quoting, again, Charles Galton Darwin:

> In dealing with the interpretation we have touched on one of the great difficulties which have made it hard to gain physical insight into the wave theory. This is the fact that *the wave equation is not in ordinary*

> *space*, *but in a co-ordinate space*, and the question arises *how this co-ordinate space is to be transcribed into ordinary space*. It would appear that most of the difficulty has arisen from *an attempt to apply it illegitimately to enclosed systems, which are really outside the idea of space*. In most of the problems we shall discuss the question hardly arises, but where it does *the correct procedure is so obvious that there is no need to deal with it in advance*. It is tempting to believe *that this will be found to be always the case* [(Darwin, 1927); *our emphasis*].

It is 'tempting', indeed, to assume that the problem will not pop up, but the evolution of physics proved that the case is quite contrary: Darwin was way too optimistic! We 'need to deal with this issue in advance', indeed: more precisely even before we start anything physical, just because the very existence of wave mechanics and quantum mechanics is conditional on the measurement. A first step is to write the Schrödinger equation in the ordinary space, and this cannot be done but only an equation for free particles, within a holographic universe [see I, §2.3, equation (2.19)].

Indeed, it would appear from the above excerpts from Darwin's work, that the idea of coordinate space is intimately connected with that of enclosed physical systems, which is 'outside', as it were, of the space concept derived from our intuition, *viz*. a particular form of ordinary space. However, inasmuch as, physically speaking, we have always to deal only with 'enclosed systems', we need either to bring the 'ordinary space' under this concept of coordinate space, or to bring this last concept under that of ordinary space. This is to be done here, as everywhere in physics for that matter, with the aid of two instruments: a *clock*, to regularize our perception of *one* physical body, and a *coordinate system*, to regularize our perception of *many* physical bodies. These are the two essential features – or *differentiae*, using a philosophical label – of a general concept of *reference frame*. The whole physics is built around this concept, and what we have to say here makes no exception.

Fact is that the Euclidean reference frame defined by us in the equation (2.3.2) is by no means special from a physical perspective. On the contrary, we have to accept that it is the most general reference frame serving any physical purpose. It was the great geometer Élie Cartan who specially insisted upon this aspect of Euclidean reference frames, brought to light by his approach of the differential geometry (Cartan, 2001). However, the issue of description of space is way deeper, mostly if this space is filled with matter. Then it becomes physical, a kind of coordinate spaces mentioned by Darwin in the excerpt above. This incident asks for some arbitrariness, and here is an excerpt from Cartan,'gauging' this arbitrariness, if we may say so, with reference frames by absolute parallelism:

> … It is easy to realize the most general way to define an *absolute parallelism* in a given Riemannian space. Attach, indeed, to the different points of this space *rectangular reference systems*, or *frames*, and this according to an *arbitrary law*; it is then sufficient *to agree* that two vectors of any origins A and B *are parallel*, or better equipollents, *if they have the same projections along the axes of reference systems* of origins A and B; *these reference systems will be then parallel themselves*. There are, therefore, in a given Riemannian space, an infinity of possible absolute parallelisms, for *the law according to which one attaches a rectangular frame to a point in space is completely arbitrary*; however, we must notice that if all the rectangular frames are rotated in the same way around their origins, one gets the same absolute parallelism; therefore, *one can define once and for all the frame attached to a particular point in space* [(Cartan, 1931); *our translation and Italics*; *see also* (Delphenich, 2011), pp. 202 – 211].

Therefore, attaching a reference frame is, indeed, according to Cartan, a *matter of gauging*: 'define once and for all the frame attached to a particular point'. However, it is quite noticeable that this frame should be taken as 'rectangular' in order to avoid further arbitrariness, so that the gauging is merely related to the orthogonal group. This kind of reference frame epitomizes the concept of coordinate space, by the classical 'box locating' of a position: as we have seen, it is sufficient to have any three numbers in order to construct an Euclidean reference frame as in equation (2.3.2), and then to orient this reference frame by parallelism. In spite of this particular choice of the frame, the Cartan definition of the absolute parallelism still remains the most general one by comparison with the definition by continuity [see (Levi-Civita, 1916)] and, what is more important, it is the only one closer to a physical spirit of definition, especially when it comes to theoretical statistical or stochastic processes calculations. Indeed, the case appears to be one of a kind, both among the ideas of Élie Cartan himself and those of differential geometry in general. As far as we are aware, this idea of definition of parallelism cannot be found, either in his previous works, or in the works following the one just cited, at least not in such an unequivocal form of expression.

One can say that the previous excerpt defines the absolute parallelism by a 'mnemonic scheme': the components of a vector, 'recorded' somehow on a memory, are reproduced by orthogonal projections in each and every one of the frames attached to positions in space according to an arbitrary, but specified, rule. In such a situation, one can say that the frames are also parallel. We find this approach to geometry closer to the spirit of modern physics, insofar as, first of all, it contains the suggestion that the definition of the frame parallelism in a given space *depends on the existence of a memory, and the possibility of* *transmitting the information* within that space. Given the fact that the classical inertia can be ascribed to the general idea of memory, one can realize the overwhelming importance of this conclusion. Secondly, Cartan's definition admits an important 'reciprocal': *we can define a class of parallel frames, once we have at our disposal three numbers physically representing the components of a vector*. We already presented an important example concerning the most important mechanism of transmitting information in space: *the propagation of light* – of a signal in general – which is the universal carrier of information in the known universe (see II, §9).

For now, let us go further into this manner of building the geometry, by showing that it is genuinely related to the *definition of torsion*. On that unique occasion, Élie Cartan insisted upon the feasibility of what, following his wording, we like to call an 'Euclidean mentality' which obviously leads to abandoning the idea of curvature, as its name would imply, but brings instead *the torsion* to the fore. According to Cartan, the torsion is contained in some kind of indecision of the vector representation in a Riemanian space and that in anentirely natural manner, as far as the Euclidean mentality is involved. Quoting:

> It is known that in the usual geometry the coordinates of a point M referred to a system of rectangular axes of origin O, are the projections of the vector $\overrightarrow{OM}$ along these axes; we can still get them joining O and M by a broken line, and then taking the sum of the projections of different parts of this line. One can even take a curved line, considered as a limit of a broken line. Now, imagine an observer located in a Riemannian space with absolute parallelism, having however an Euclidean mentality. If this observer, placed in O and adopting a system of rectangular axes of origin O, *wants to calculate the coordinates which he must assign to a point* M (*our Italics*), he will join O with M by a continuous line, and will proceed as we just have shown: he will consider the line OM as a geometric sum of a very large number of minute vectors; he will transport them in O parallel with themselves, and then will take their geometric sum: thus he will find a vector of origin O, which he will consider as equipollent to the line OM, and whose *projections upon axes shall be the coordinates he sought for* (*our Italics*). If the observer joins O and M by another line, he will be led to consider it as equipollent to a second vector, which *generally will not be the same with the first vector*. In other words, the different lines joining O and M are not all equipollent to the same vector.
>
> The issues can be presented yet another way. If in the Euclidean geometry one considers a closed contour, or cycle C, pursued in a certain direction, it is equipollent to a null vector, according to a fundamental theorem of the vector calculus; in a Riemannian space with absolute parallelism this is no more the case: the cycle C is equipollent to a certain

vector, which we shall call *torsion vector*. Only in the Euclidean space we will have, for all cycles, null torsion vector [(Cartan, 1931); *our translation, Italics in the original, except as mentioned*; *see also* (Delphenich, 2011), *loc. cit.*].

As an observation connected to this definition of the torsion, we have the concept of matter that can emerge from a quotidian example: that of the Earth as a planet. First, let us apply the above idea of Cartan to an operational definition of the vertical direction on Earth surface accessible to our regular displacements. This, of course, will get us some unit vectors to be represented as points on the unit sphere. With *any* three of these directions, we can construct an estimator of the position of center of Earth, *viz*. the point toward which the force of weight of earthly bodies presumably acts. This center is, nonetheless, *never unique*, but varies within a region inside the Earth, to which we never have access, for the space itself has never access there. According to Cartan's idea this region has a finite extension which can be given by a torsion vector. In general, the matter of Earth's nucleus – even more general, of a spatially extendedparticle – is described by the torsion vector. In physics, this idea made its way only in our times (Delphenich, 2013), and it is related to a *concept of density* able to properly generalize the Newtonian concept in order to allow for a scale transition physical theory of the universes.

REFERENCES

Aharonov Y., Anandan J., Vaidman L., *Meaning of the Wave Function*, Physical Review A, **47**, *6*, 4616-4626 (1993).

de Broglie L., (a) *Sur le Parallélisme entre la Dynamique du Point Matériel et l'Optique Géométrique*, Le Journal de Physique et le Radium, Série VI, **7**, *1*, 1-6; (b) *Sur la Possibilité de Relier les Phénomènes d'Interférence et de Diffraction à la Théorie des Quanta de Lumière*, Comptes Rendus de l'Académie des Sciences de Paris, **183**, 447-448; (c) *Interference and Corpuscular Light*, Nature, **118**, 441-442; (d) *Les Principes de la Nouvelle Mécanique Ondulatoire*, Le Journal de Physique et le Radium, **7**, *10*, 321-337; (e) *Ondes et Mouvements*, Gauthier-Villars, Paris (1926).

Cartan E., *Notice Historique sur la Notion de Parallélisme Absolu*, Mathematische Annalen, **102**, 698-706 (1930).

Cartan E., *Le Parallélisme Absolu et la Théorie Unitaire du Champ*, Revue de Métaphysique et de Morale, **38**, *1*, 13-28 (1931).

Cartan E., *Riemannian Geometry in an Orthogonal Frame*, World Scientific Publishing, Singapore (2001).

Clausius R., *On a Mechanical Theorem Applicable to Heat*, Philosophical Magazine, **40**, *265*, 122-127 (1870).

Darwin C.G., *Free Motion in the Wave Mechanics*, Proceedings of the Royal Society of London Series A, **117**, 258-293 (1927).

Delphenich D.H., *Selected Papers on Teleparallelism*, www.neo-classical-physics.info (2011).

Delphenich D.H., (a) *A Strain Tensor that Couples to the Madelung Stress Tensor*, arXiv:1303.3582v1 [quant-ph]; (b) *The Use of Teleparallelism Connection in Continuum Mechanics*, arXiv: 1304.16477v1 [gr-qc] (2013).

Drukarch B., Holland H.A., Velichkov M., Geurts J.J.G., Voorn P., Glas G., de Regt H.W. *Thinking about the Nerve Impulse: a Critical Analysis of the Electricity-Centered Conception of the Nerve Excitability*, Progress in Neurobiology, **169**, 172-185 (2018).

Ernst F.J., *New Formulation of the Axially Symmetric Gravitational Field Problem I*, Physical Review, **167**, 1175-1178; *New Formulation of the Axially Symmetric Gravitational Field Problem II*, Ibidem, **168**, 1415-1417 (1968).

Ernst F.J., *Exterior Algebraic Derivation of Einstein Field Equations Employing a Generalized Basis*, Journal of Mathematical Physics, **12**, 2395-2397 (1971).

Fixsen D.J., Cheng E.S., Gales J.M., Mather J.C., Shafer R.A., Wright E.L. *The Cosmic Microwave Background Spectrum from the Full COBE FIRAS Data Set*, The Astrophysical Journal, **473**, 576 -587 (1996).

Frederick C., *Stochastic Space-Time and Quantum Theory*, Physical Review D, **13**, *11*, 179-196 (1976).

Fresnel A., *Considérations Mécaniques sur la Polarisation de la Lumière*, Annales de Chimie et de Physique, **17**, 179-196 (1821); reprinted in Oeuvres Complètes, Imprimerie Impériale, Paris 1866, Tome Premier, 629-645 (can be found at Gallica.bnf.fr).

Fresnel A., *Mémoire sur la Diffraction de la Lumière*, Mémoirs de l'Académie des Sciences de l'Institute de France, **5**, 339-475 (1826); reprinted in Oeuvres Complètes, Imprimerie Impériale, Paris 1866, Tome Premier, 247-382 (can be found at Gallica.bnf.fr).

Fresnel A., *Mémoire sur la Double Réfraction*, Mémoirs de l'Académie des Sciences de l'Institute de France, **7**, 45-176 (1827); reproduced in Oeuvres Complètes, Imprimerie Impériale, Paris, 1866.

Gauss C.F., *Théorèmes Généraux sur les Forces Attractives et Répulsives qui Agissent en Raison Inverse du Carré des Distances*, Journal des Mathématiques Pures et Appliquées (Journal de Liouville), **7**, 273-324 (1842).

Georgescu-Roegen N., *On the Texture of the Arithmetical Continuum*, Appendix A in *The Entropy Law and the Economic Process*, Harvard University Press, Cambridge MA, USA (1971).

Hertz H., *The Principles of Mechanics, Presented in a New Form*, Dover Phoenix Editions (2003).

Hobbes T., *Tractatus Opticus* (1644), in *Thomae Hobbes Malmesburiensis Opera Philosophica Quae Latine Scripsit Omnia*, **5**, 215-248, J. Bohn, London, 1839.

Hooke R., *Micrographia, or Some Physiological Descriptions of Minute Bodies Made by Magnifying Glasses*, Martyn & Allestry, London (1665).

Jackson J.D., *Classical Electrodynamics*, John Wiley & Sons, Inc., New York (1998).

Kemble E.C., *The Fundamental Principles of Quantum Mechanics*, McGraw-Hill Book Company, Inc., New York & London (1937).

Langevin P., *Sur l'Origine des Radiations et l'Inertie Électromagnétique*, Journal de Physique Théorique et Appliquée, **4**, *1*, 165-183 (1905).

Levi-Civita T., *Nozione di Parallelismo in una Varietà Qualunque...*, Rendiconti del Circolo Matematico di Palermo, **42**, 173-204 (1916).

Maxwell J.C., *Scientific Papers*, Edited by W. D. Niven, Dover Publications (1965).

Mazilu N., *Black-Body Radiation Once More*, Bul. Inst. Polit. Iaşi, **LVI(LX)**, *2*, 69-97 (2010).

Mazilu N., *Physical Principles in Revealing the Working Mechanisms of Brain, Part I*, Bul. Inst. Polit. Iaşi, **65**(**69**), *4*, 17-55 (2019).

Mazilu N., *Physical Principles in Revealing the Working Mechanisms of Brain, Part II*, Bul. Inst. Polit. Iaşi, **66**(**70**), *1*, 33-69 (2020).

Mazilu N., Agop M., Mercheș I., *The Mathematical Principles of Scale Relativity Physics*, Taylor & Francis Group, London, New York (2019).

Mazilu N., Agop M., Mercheș I., *Scale Transitions as Foundation of Physics*, World Scientific, Singapore (to appear soon) (2020).

Mihăileanu N., *Complements of Analytic, Projective and Differential Geometry*, Editura Didactică şi Pedagogică, Bucureşti, Romania (in Romanian) (1972).

Newton Sir Isaac, *Opticks, or a Treatise of the Reflections, Refractions, Inflections & Colours of Light*, Dover Publications, Inc., New York (1952).

Nottale L., *Scale Relativity and Fractal Space-Time*, Imperial College Press, London, UK (2011).

Poincaré H., *Remarques sur une Expérience de M. Birkeland*, Comptes Rendus de l'Académie des Sciences de Paris, **123**, 530-533 (1896).

Poincaré H., *Les Idées de Hertz sur la Mécanique*, Revue Générale des Sciences Pures et Appliquées, **8**, *18*, 734-743 (there is an English translation at vixra.org, and also one by David Delphenich) (1897).

Poincaré H., *Sur la Dynamique de l'Électron*, Rendiconti del Circolo Matematico di Palermo, **21**, 129-176 (1906).

Pribram K.H., *Thoughts on the Meaning of Brain Electrical Activity*, Internat. Journal of Psychology, **33**, *3*, 213-225 (1998).

Pribram K.H., *Holonomic Brain Theory*, Scholarpedia, **2**, *5*, 2735 (2007).

Resnikoff H.L., *Differential Geometry of Color Perception*, Journal of Mathematical Biology, **1**, 97-131(1974).

Sciama D.W., *The Physical Foundations of General Relativity*, Doubleday, New York (1969).

Schrödinger E., *Grundlinien einer Theorie der Farbenmetrik im Tagessehen*, Annalen der Physik, Band **63**(21), 397-426; 427-456; **63**(22), 481-520 (1920); English translations by Keith K Niall, in the collection (Niall, 2017).

Shapiro A.E., *Kinematic Optics: A Study of the Wave Theory of Light in the Seventeenth Century*, Archives for the History of Exact Sciences, **11**, 134-266 (1973).

PRINCIPII FIZICE ÎN EVIDENȚIEREA MECANISMELOR DE FUNCȚIONARE A CREIERULUI. PARTEA A III-A

(Rezumat)

În prezenta lucrare creierul este modelat fizic ca un univers, analog cosmologiilor standard. În timp ce în modelul fizic domină gravitaţia, în universul creierului domină electromagnetismul, descrierea matematică în cele două cazuri fiind asemănătoare. Apelând la descrierea metrică a materiei, se tratează imaginea spaţiului clasic în strânsă corelaţie cu memoria creierului, toate acestea fiind fundamentate pe baza conceptului de inerţie.